

RC Circuit

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1 Theory

1.1 RC Circuit

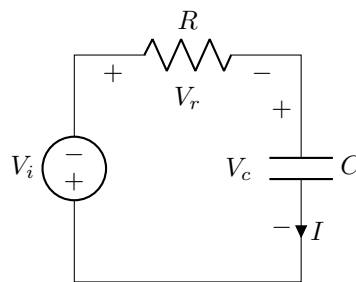


Figure 1: RC circuit

We consider the RC circuit (see fig. 1) as a system with the voltage V_i as the input and the voltage across the capacitor V_c as the output. At the capacitor

$$Q = CV_c,$$

where C is the capacity (a constant) and Q is the charge stored in the capacitor. At the resistor (Ohms law)

$$RI = V_r,$$

where V_r is the voltage across the resistor and R is the resistance (a constant). The loop rule implies that

$$V_i = V_c + V_r.$$

The current rule implies that the current I at the capacitor is the current at the resistor. The current at the capacitor is the derivative of the stored charge. Therefore combining the equations:

$$V_i = Q/C + R\dot{Q}.$$

This can be written in the standard form with Q as the state variable:

$$\dot{Q} = -Q/(RC) + V_0/R.$$

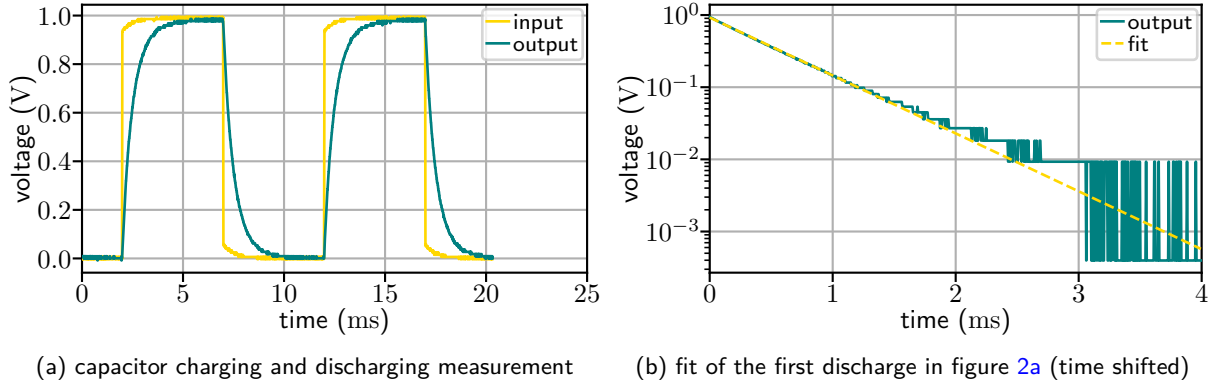


Figure 2: capacitor charging and discharging

Therefore the RC circuit is equivalent to the system $(\mathcal{A}, \mathcal{B}, \mathcal{C})$, where

$$\mathcal{A} := -1/(RC), \quad \mathcal{B} := 1/R, \quad \mathcal{C} := 1/C.$$

From now on system theory notation, that is for $u \in L^1_{loc}$ and $x_0 \in \mathbb{R}$ we denote by $x(\bullet, x_0, u)$ the state of the system (that is the charge of the capacitor as a function of time) with initial charge x_0 and input voltage u .

In the case of no input: $\forall t \in [0, \infty)$ and $x_0 \in \mathbb{R}$:

$$x(t, x_0, 0) = x_0 \exp(-t/(RC)).$$

1.2 Transfer function

The transfer function G is given by

$$G(s) = \mathcal{C}(sI - \mathcal{A})^{-1}\mathcal{B} = \frac{1}{RC} \frac{1}{s + 1/(RC)} = \frac{1}{1 + \omega_0^{-1}s},$$

where $\omega_0 := 1/(RC)$. Let $\omega \in (0, \infty)$, $G(i\omega) \neq 0$, $u_0 \in \mathbb{R}$ and $u(t) := u_0 \sin(\omega t)$. Let $r := |G(i\omega)|$ and $\varphi := \arg G(i\omega)$. Then asymptotically (for $t \rightarrow \infty$) the output of the system with input u and arbitrary initial value is given by y_a , where

$$y_a(t) := u_0 r \sin(\omega t + \varphi).$$

Now

$$|G(i\omega)| = \frac{1}{|1 + RCi\omega|} = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}$$

and

$$\arg(G(i\omega)) = -\arctan(\omega/\omega_0).$$

2 Experiment

Used $C = 10^5$ pF and $R = 1.6 \times 10^3 \Omega$ (including the output impedance of the signal generator). Therefore we expect $\tau := RC \approx 1.6 \times 10^{-4}$ s. The input voltage can be set with a signal generator that is part of the oscilloscope. The input and the output (the voltage across the capacitor) of the system are monitored with an oscilloscope, which can log the voltage as a function of time.

2.1 Discharge Curve

To obtain τ experimentally: We charge the capacitor first by applying a constant 1 V input. The input is then set to 0 and the output is measure for some time. In practice this is implemented by using as input a periodic step function with large period compared to τ . The exponential decay law can then be fitted to the discharge data. The result of the fit is $\tau = (5.41 \pm 0.01) \times 10^{-4}$ s. The measured data and the fit are visualised in figure 2.

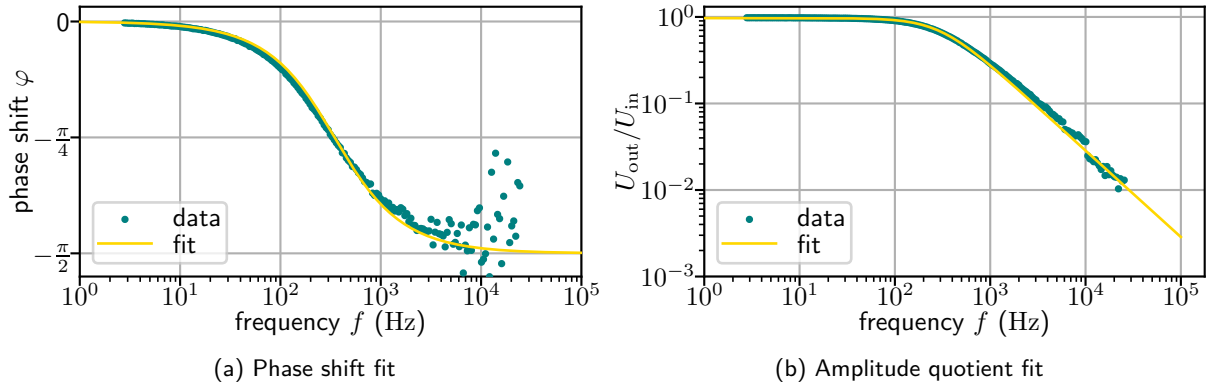


Figure 3: Transfer function fit

2.2 Transfer Function

Let $f_0 := 2\pi/\tau$. For $f \in (0, \infty)$ and $U_{\text{in}} = 2\text{V}$ we apply

$$U_{\text{in}} \sin(2\pi f \bullet)$$

as input. We wait for 2s so that the transient settles and then measure the output. From the theory section: The output has the (asymptotic) form

$$U_{\text{out}} \sin(2\pi f \bullet + \varphi),$$

where

$$U_{\text{out}} = \frac{1}{\sqrt{1 + f/f_0}} U_{\text{in}}, \quad \varphi = -\arctan(f/f_0).$$

To obtain U_{out} and φ we do a least squares fit of the output signal. We then repeat this procedure for different frequencies and fit the above relation (with f_0 as the parameter). The results can be found in figure 3. For the phase shift fit the optimal τ was $\tau = (4.6 \pm 0.2) \times 10^{-4}$ s. For the amplitude quotient fit the optimal τ was $\tau = (5.39 \pm 0.03) \times 10^{-4}$ s.